

# Search for Dynamical Symmetry Breaking Physics by Using Top Quark\*

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## Abstract

We report first results from the investigation on probing dynamical mechanism of the electroweak symmetry breaking using top quark. We consider the case where the top mass originates from a fermion-antifermion pair condensate, which necessitates (i)strong interaction to cause condensation, and (ii)4-fermi interaction to give top mass. From the observed top mass and the unitarity constraint, we obtain, for the 4-fermi interaction (ii), lower bounds for its strength and upper bounds for its intrinsic new mass scale as we vary the type of the strong interaction (i). Bethe-Salpeter and Schwinger-Dyson equations are solved numerically to study the dynamical symmetry breaking effect semi-quantitatively.

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## 1. Introduction

The  $SU(2) \times U(1)$  gauge theory for describing the electroweak interactions has been very successful both theoretically and experimentally. However, all the experimental tests have been done only for its gauge part and we have little knowledge on the electroweak symmetry breaking mechanism so far. In the light of the naturalness problem, we may suppose that there exists some new physics related to the electroweak symmetry breaking at the energy scale  $O(100\text{GeV} - 1\text{TeV})$ . Dynamical symmetry breaking is one of the attractive candidates for the solution to the naturalness problem. We consider this possibility and intend to study how to search dynamical symmetry breaking physics at future colliders.

Here we use top quark as a probe for the symmetry breaking sector. The top quark, being much heavier than other fermions with the mass of the order of the electroweak symmetry breaking scale [1], is expected to couple most strongly to the symmetry breaking sector. In this paper, we assume the gauge symmetry is broken dynamically by a fermion-antifermion pair condensate and that the top quark acquires its mass  $m_t$  through a 4-fermi interaction

$$\mathcal{L} = \frac{1}{\Lambda^2} \bar{\Psi}_L \Psi_R \bar{t}_R t_L + h.c. , \quad (1)$$

where  $\Lambda$  represents the new physics scale and  $\Psi$  denotes a new fermion introduced in the symmetry breaking sector. Then  $m_t$  is estimated to be

$$m_t = \frac{\langle \bar{\Psi}_L \Psi_R \rangle}{\Lambda^2}. \quad (2)$$

On the other hand, since the condensate  $\langle \bar{\Psi}_L \Psi_R \rangle$  has a non-zero  $SU(2)$  charge, it also contributes to the gauge bosons' masses. Thus we naively expect a following inequality:

$$\langle \bar{\Psi}_L \Psi_R \rangle^{1/3} \leq \Lambda_{EW} \sim O(250 \text{ GeV}), \quad (3)$$

where  $\Lambda_{EW}$  represents the electroweak symmetry breaking scale. We have inequality for there may exist other fermion condensations which are  $SU(2) \times U(1)$  non-invariant. From the observed value of top mass  $m_t \simeq 175 \text{ GeV}$  [1], this naive argument suggests the 4-fermi interaction (1) is rather strong and is bounded from below  $1/\Lambda^2 \geq m_t/\Lambda_{EW}^3$  due to eqs.(2) and (3). It means that the new physics scale  $\Lambda$  is not too far from  $\Lambda_{EW}$ . This is the reason why we use the top quark (or the 4-fermi interaction (1)) to probe the symmetry breaking mechanism. From the non-standard effect to e.g.  $Z\bar{t}t$  vertex as shown in fig.1, we would be able to study dynamical symmetry breaking physics at future colliders.

In this note, we report, as our first results, the numerical estimation of a 4-fermi interaction which gives rise to the top mass dynamically. In this analysis, we assume a strong interaction which causes the fermion and anti-fermion pair condensate, and a 4-fermi interaction to give the top mass. For the strong interaction, we examine the non-abelian gauge interaction cases such as  $SU(2)$  or  $SU(3)$  and the fixed coupling constant case. From the experimental value of  $m_t$  and the unitarity constraint, we obtain lower bounds for the strength of the 4-fermi interaction and upper bounds for its intrinsic new mass scale.

## 2. Numerical Estimation of the 4-Fermi Interaction

First, we explain the electroweak symmetry breaking sector considered here. We introduce fermions to the breaking sector following the one-doublet technicolor (TC) model[2, 3],

$$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L, \quad U_R, \quad D_R, \quad (4)$$

with the weak hypercharge,

$$Y(Q_L) = 0, \quad Y(U_R) = \frac{1}{2}, \quad Y(D_R) = -\frac{1}{2}, \quad (5)$$

where these quantum numbers are chosen to cancel anomaly.  $N_d$  sets of above new fermions are introduced in the breaking sector.

In order to characterize the strong interaction between the above fermion and antifermion, we introduce kernel  $K$ . For example, in the TC models where the  $SU(N_{TC})$  gauge interaction cause the condensation, we can approximately write down the explicit form of the kernel  $K$ . In the improved ladder approximation in Landau gauge [4], it is written in the momentum space as [5] (momentum configuration is defined in fig.2)

$$K(p, q) = C_2 g^2(p, k) (\gamma^\mu \otimes \gamma^\nu) \frac{1}{(p-k)^2} \left[ g_{\mu\nu} - \frac{(p-k)_\mu (p-k)_\nu}{(p-k)^2} \right], \quad (6)$$

where  $C_2$  is the second Casimir invariant of the  $SU(N_{TC})$  fundamental representation and  $g(p, k)$  is the running coupling constant. We can deal with various types of strong interaction by changing the form of the kernel  $K$ . Here we consider two possibilities. One is the non-abelian gauge interaction ( $SU(2)$  and  $SU(3)$ ) case and the other is the fixed coupling case.

Once the strong interaction to cause a condensate is chosen and the form of the kernel is fixed, we can write down the SD eq. and BS eq. These equations in the ladder approximation are shown diagrammatically in fig.3. By solving these equations numerically, we obtain the full propagator of  $U$  and  $D$ , and the decay constant  $F_\pi$ , and so on [5, 6]. Note that this approach have been applied to for the QCD case and the results of numerical calculation are in good agreement with the experimental values [5, 6] <sup>\*</sup>.

**Top Quark Mass** Here we calculate the top mass following the approach discussed above. We introduce a 4-fermi interaction which induces the top mass,

$$\mathcal{L} = \frac{G}{M^2} (\overline{Q}_L U_R) (\overline{t}_R q_L) + h.c. , \quad (7)$$

where  $q_L$  denotes the ordinary quark weak doublet and  $G$  the dimensionless coupling. Because the 4-fermi interaction (7) cannot be a fundamental interaction, there should be a mass scale above which this interaction will resolve. We call this new physics scale  $M$ . Therefore the 4-fermi interaction (7) should be regarded as an effective interaction which is appropriate below energy scale  $M$  <sup>†</sup>.

Now the top quark acquires its mass through the 4-fermi interaction (7) as shown in fig.4. Using the full propagator of  $U$  which is a solution to the SD equation <sup>‡</sup>, the top mass can be calculated as

$$m_t = \frac{G}{M^2} \langle \overline{U} U \rangle_M , \quad (8)$$

where

$$\begin{aligned} \langle \overline{U} U \rangle_M &= \frac{1}{2} \int^M \frac{d^4 p}{(2\pi)^4} \text{tr} \left( \frac{i}{\not{p} - \Sigma(p)} \right) \\ &= \frac{1}{2} \frac{N_{TC} N_d}{4\pi^2} \int^{M^2} dx \frac{x \Sigma(x)}{x + \Sigma(x)^2} \end{aligned} \quad (9)$$

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<sup>\*</sup> Taking  $f_\pi$  as the only input parameter,  $\Lambda_{QCD}$ ,  $\langle \overline{\Psi} \Psi \rangle$ , and various meson masses and decay constants have been calculated in refs.[5, 6]. So we expect to calculate the top mass, the oblique corrections, the  $Z\bar{t}t$  vertex, etc., semi-quantitatively.

<sup>†</sup> For the models such as the extended TC model[7], various 4-fermi interaction other than (7) are induced below energy scale  $M$ . We do not consider such a possibility. In this article we only introduce the 4-fermi interaction (7) by hand.

<sup>‡</sup> In fact, the SD eq. shown in fig.3 is incomplete. Because of the 4-fermi interaction (7), we should consider the mixing of SD eqs. of top quark and  $U$ , but here we neglect this effect.

with  $x = p_E^2 = -p^2$ . Because above the scale  $M$  the 4-fermi interaction will resolve, we set the upper bound of the integral as  $M$  <sup>§</sup>.

Using the decay constant  $F_\pi$  <sup>¶</sup> which is obtained by solving the BS eq., we can set the mass scale of the theory. From the gauge bosons masses,  $F_\pi$  is normalized as

$$M_W \geq \frac{gF_\pi}{2}, \quad (10)$$

where  $g$  is the  $SU(2)_L$  gauge coupling constant. Here we have inequality rather than equality, because there may be fermion condensates other than  $\langle \bar{U}U \rangle$  and  $\langle \bar{D}D \rangle$  which do not contribute to the top mass while giving masses to the gauge bosons.

By substituting the observed top mass  $m_t \simeq 175$  GeV [1] in eq.(8), we obtain the coupling  $G$  for a given  $M$ . We consider the case where the strong interaction is given by the  $SU(2)$  and  $SU(3)$  gauge interaction, and the constraints on  $G$  are given as the function of  $M$  in figs.5 and 6, respectively. In each figures, the upper region of the curved lines are allowed because of the inequality in eq.(10). The coupling  $G$  is found to be bounded from below.

The behavior of the lower bound of  $G$  can be understood by considering the momentum dependence of the mass function  $\Sigma(x)$ . For  $x \ll \Lambda_{TC}^2$  ( $\Lambda_{TC}$  is defined as the scale where the leading-logarithmic running coupling constant diverges),  $\Sigma(x)$  is almost flat. This gives the asymptotic behavior of the lower bound as a function of  $M$ ,

$$G \sim \frac{1}{M^2}. \quad (11)$$

The lower bound of  $G$  increases rapidly as  $M \rightarrow 0$ , because the upper bound of the integral in eq.(9) becomes smaller. However, we neglect this region of  $M$  ( $M \ll \Lambda_{TC}$ ) because our effective treatment of the 4-fermi interaction (7) becomes invalid. On the other hand, for  $x \gg \Lambda_{TC}^2$ , the mass function decreases rapidly as

$$\Sigma(x) \sim \frac{1}{x} (\ln x)^{\frac{1}{4B}-1}, \quad (12)$$

where  $B = \beta_0/12C_2$  with  $\beta_0$  being the lowest order coefficient of the  $\beta$  function. From this momentum dependence, the lower bound behaves as

$$G \sim \frac{M^2}{(\ln M^2)^{\frac{1}{4B}}}. \quad (13)$$

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§ In fact the new physics scale  $M$  is defined as the upper bound of this integral in this article.

¶ We define the decay constant  $F_\pi$  as  $\langle 0 | \bar{\Psi} T^a \gamma^\mu \gamma^5 \Psi | \pi^a(p) \rangle = -ip^\mu F_\pi$ , where  $\Psi = (U, D)$ .

Therefore the lower bound for coupling  $G$  is found to be approximately proportional to  $M^2$  and the lower bound for the strength of the 4-fermi interaction (7), typically  $\sim G/M^2$ , is obtained. This result agrees with the naive discussion given in the introduction.

We also consider the fixed coupling constant case (  $B = 0$  ) in order to apply to the QED like theory or walking technicolor theory[8]. In fig.7 we show the constraints for the coupling  $G$ . In this case, as  $x \rightarrow \infty$ , the decreasing behavior of  $\Sigma(x)$  is different from the former case, and is

$$\Sigma(x) \sim \frac{1}{\sqrt{x}}. \quad (14)$$

From this dependence the lower bound of the coupling  $G$  increases linearly as the function  $M$  and the lower bound of the strength is weaker than the one in former non-abelian gauge interaction cases.

**Unitarity Constraint** Next, we consider the unitarity constraint and put upper bounds for  $G$ . First of all, we explain the unitarity bounds which are used here. Let us consider a two-body to two-body scattering (  $1 + 2 \rightarrow 3 + 4$  ). We take the helicity of each particle as  $\lambda_1, \lambda_2, \lambda_3$ , and  $\lambda_4$ . In the center of mass (C.M.) frame the partial-wave expansion for the helicity amplitude is,

$$M_{fi} = \frac{8\pi}{\sqrt{\beta_i \beta_f}} \sum_0^\infty (2J+1) T_{\lambda_1, \lambda_2; \lambda_3, \lambda_4}^J(\sqrt{s}) d_{\lambda_i, \lambda_f}^J(\theta) e^{i(\lambda_i - \lambda_f)\phi}, \quad (15)$$

where  $\lambda_i = \lambda_1 - \lambda_2$ ,  $\lambda_f = \lambda_3 - \lambda_4$ ,  $\beta_i = 2|\mathbf{p}_i|/\sqrt{s}$ ,  $\beta_f = 2|\mathbf{p}_f|/\sqrt{s}$  and  $d_{\lambda_i, \lambda_f}^J(\theta)$  is the Wigner's  $d$ -function. And  $\mathbf{p}_i(\mathbf{p}_f)$  is the common C.M. momentum for the initial (final) state and  $\sqrt{s}$  the C.M. energy. The angle  $(\theta, \phi)$  is taken as the direction angle of the final particle 3 measured from the direction of the initial particle 1. From the unitarity condition for the  $S$ -matrix  $S^\dagger S = SS^\dagger = 1$ , the unitarity bounds for the coefficient of the partial-wave expansion  $T_{\lambda_1, \lambda_2; \lambda_3, \lambda_4}^J(\sqrt{s})$  are obtained as

$$|\text{Re} T^J| \leq 1 \quad (\text{for an elastic channel}) \quad (16)$$

$$|T^J| \leq 1 \quad (\text{for an inelastic channel}) \quad (17)$$

Using these unitarity bounds, we put the upper bound of  $G$ .

We consider the two-body to two-body scattering of fermions through the 4-fermi interaction (7) at the energy scale  $E \gg \Lambda_{TC}$  where the effect of confinement can be neglected.

Matrix elements of these processes grow as  $\sim E^2$  as the scattering energy increases, and it would break the above unitarity bounds. But above the energy scale  $M$ , the 4-fermi interaction should resolve to retain unitarity. Thus the upper bound for  $G$  is obtained by assuming that the unitarity condition would not break down below the scale  $M$ .

We found that the most stringent bound comes from the scattering process  $t\bar{t} \rightarrow U\bar{U}$ . From the 4-fermi interaction (7), this process occurs only in a scalar channel ( $J=0$ ). The coefficient of the partial-wave expansion in the massless limit  $m_t = m_U = 0$  is given by

$$T^0(\sqrt{s}) = \frac{1}{8\pi} \frac{G s}{M^2}. \quad (18)$$

By the above assumption, the upper bound for the coupling  $G$  comes from

$$|T^0(\sqrt{s} = M)| \leq 1, \quad (19)$$

and this leads,

$$G \leq 8\pi. \quad (20)$$

This bound for the coupling  $G$  is shown in figs. 5, 6, and 7.

Combining the previous estimation of the top mass and this unitarity constraint, we obtain the allowed region in the  $G$ – $M$  plane for each cases. From these figures the typical value of the coupling  $G/4\pi$  is found to be  $O(1)$  in all cases and to be a rather strong coupling constant. Moreover, we can find the upper bounds for the new physics scale  $M$  in the 4-fermi interaction (7). For the  $SU(2)$  and  $SU(3)$  gauge interaction case, the upper bounds are around 5 TeV. For the fixed coupling constant case, the bound is much weaker than the former case and is around 15 TeV.

### 3. Conclusions and Discussion

We have considered the case where the top quark acquires mass from a fermion-antifermion condensate. In this case, we assume a strong interaction to cause condensation and a 4-fermi interaction to give top mass. We consider two possibilities for the strong interaction. One is the non-Abelian gauge interaction (  $SU(2)$  and  $SU(3)$  ) case. The other is the fixed coupling constant case. From the observed top quark mass and the unitarity constraint, we obtain the following results for the 4-fermi interaction in each case.

- Lower bound for its strength is obtained.
- Coupling  $G/4\pi$  is typically of  $O(1)$ .
- Upper bound for the new physics scale  $M$  is obtained:
  - $M \lesssim 5$  TeV for the non-Abelian gauge interaction case.
  - $M \lesssim 15$  TeV for the fixed coupling case.

The above results will be the bases in studying the dynamical symmetry breaking mechanism by using top quark. From these results, we intend to calculate the non-standard corrections to the  $\gamma\gamma\bar{t}t$  vertex, the  $Z\bar{t}t$  vertex, the  $W\bar{t}b$  vertex and so on. Our goal will be to discuss the detectability of these corrections at future  $e^+e^-$  and hadron colliders, and probe the dynamical symmetry breaking mechanism by using top quark. This will be given elsewhere[9].



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## Figure Caption

- Figure 1: Diagram representing the non-standard correction to the  $Z\bar{t}t$  vertex from the dynamical symmetry breaking sector.
- Figure 2: Feynman diagram for the kernel  $K(p, k)$ . The helix denotes the technigluon propagator.
- Figure 3: The graphical representations of the SD eq. and the BS eq. in the ladder approximation. The line with a blob denotes the full propagator of  $\Psi$ .
- Figure 4: The diagram for the top quark mass.
- Figure 5: Allowed region in the  $G$ - $M$  plane for  $m_t = 175$  GeV. The  $SU(2)$  gauge interaction is considered as the strong attractive force to cause the condensation. The lower bound for  $G$  from the top quark mass is shown with curved lines in the case where the number of the set of  $U$  and  $D$ ,  $N_d$  is taken as  $N_d = 1, 2$ , and  $3$ . In this case,  $\Lambda_{TC} = 1.7, 1.1$ , and  $0.7$  TeV for  $N_d = 1, 2$ , and  $3$ , respectively. The upper bound for  $G$  from the unitarity constraint is also shown. From this figure, we can obtain the upper bounds for the scale  $M$ .
- Figure 6: Allowed region in the  $G$ - $M$  plane for  $m_t = 175$  GeV. The  $SU(3)$  gauge interaction is considered as the strong attractive force to cause the condensation. The lower bound for  $G$  from the top quark mass is shown with curved lines in the case where the number of the set of  $U$  and  $D$ ,  $N_d$  is taken as  $N_d = 1, 2$ , and  $3$ . In this case,  $\Lambda_{TC} = 1.3, 0.9$ , and  $0.7$  TeV for  $N_d = 1, 2$ , and  $3$ , respectively. The upper bound for  $G$  from the unitarity constraint is also shown. From this figure, we can obtain the upper bounds for the scale  $M$ .

Figure 7: Allowed region in the  $G$ - $M$  plane for  $m_t = 175$  GeV. The interaction of a fixed coupling constant considered as the strong attractive force to cause the condensation. The lower bound for  $G$  from the top quark mass is shown with curved lines in the case where the number of the set of  $U$  and  $D$ ,  $N_d$  is taken as  $N_d = 3, 6$ , and  $9$ . The upper bound for  $G$  from the unitarity constraint is also shown. From this figure, we can obtain the upper bounds for the scale  $M$ .